INTERNATIONAL SCHOOL OF SUBNUCLEAR PHYSICS

53rd Course: THE FUTURE OF OUR PHYSICS INCLUDING NEW FRONTIERS

Erice 24 June - 3 July 2015

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Relaxation of a non Abelian plasma: a holographic model

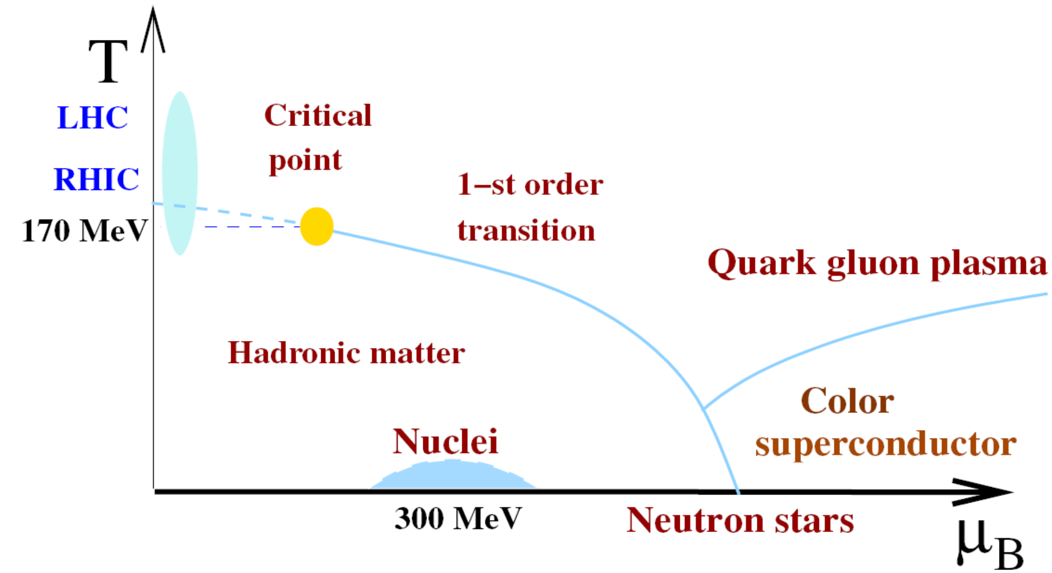
based on arXiv:1503.01977 (to appear on JHEP)

work with Pietro Colangelo, Fulvia De Fazio and Floriana Giannuzzi

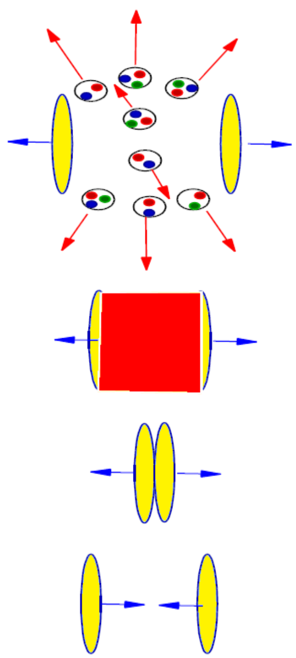
Relaxation of a far-from equilibrium QGP

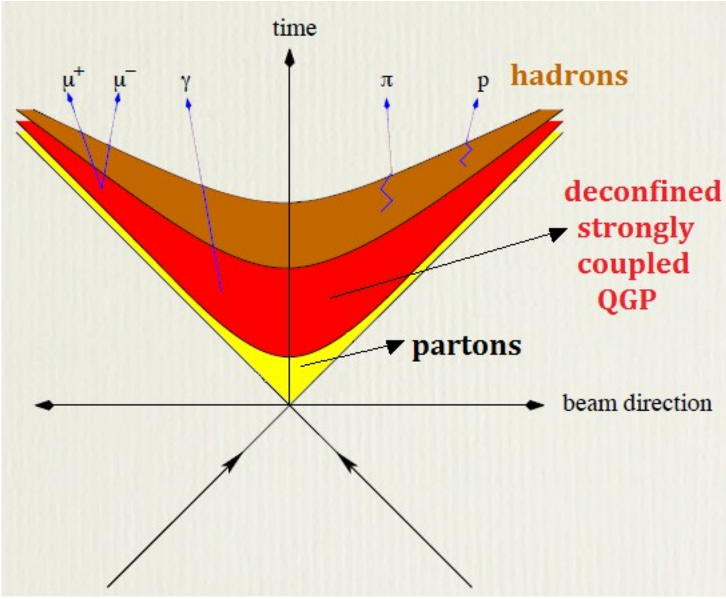
Quark-Gluon Plasma:

a new state of matter formed from deconfined quarks, antiquarks and gluons at high T



Physical picture of QGP formation in Heavy Ion Collisions (LHC, RHIC)





OUR FOCUS:
evolution of the QGP from a pre-equilibrium state and estimate of physical observables (effective temperature, entropy density, energy density, pressure)

QGP as a strongly coupled fluid

Evidence from the RHIC and LHC experiments: onset of the hydrodynamic regime for time scales larger than 1 fm/c after the collision.

WHAT ABOUT THE PRE-EQUILIBRIUM EVOLUTION?

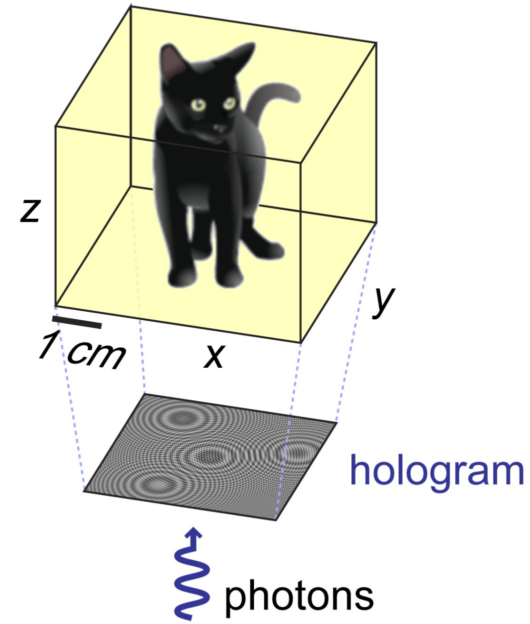
The QGP produced is a strongly-coupled fluid which exhibits collective behavior.

⇨ **Perturbative methods inapplicable!**

HOLOGRAPHIC THERMALIZATION: AdS/CFT correspondence as a tool to describe non-perturbative dynamics of QGP.

Holography:
an optical technology by which a three-dimensional image is stored on a two dimensional surface via a diffraction pattern

Holographic principle ('t Hooft, Susskind):
states in a spacetime region can equally well be represented by bits of information contained in its surface boundary



The AdS/CFT correspondence

Strongly-coupled

Conformal **F**ield **T**heory

on the Minkowski space

$\mathcal{M}_4$

A conformal compactification gives

the *Boundary* of

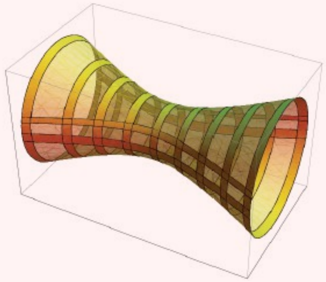
duality

Wealky-coupled gravitational string theory on

$AdS_5 \times S^5$

Anti-de Sitter hyperboloid

Bulk



Quantum field theory on $\mathcal{M}_4$

at finite stationary T

duality

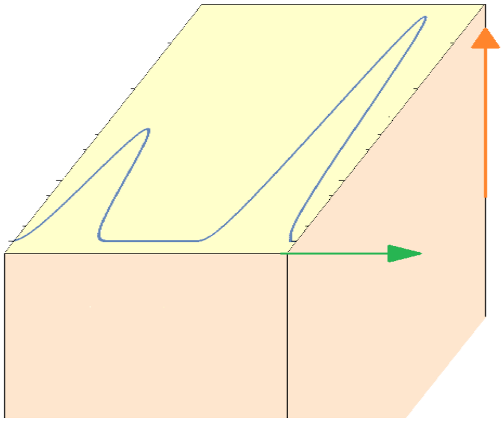
AdS_5 / BH metric

Black **H**ole: horizon → T

QGP formation and relaxation in holography

BOUNDARY SOURCING APPROACH

a time-dependent deformation pulse (quench) is introduced to the metric on the boundary in order to mimic the effects of heavy ion collisions.



QGP evolution towards equilibrium is computed in the 5-dimensional dual space from Einstein equations.

1 $\mathcal{M}_4$ + quench deformation pulse

duality →

2 5d curved metric driven out-of-equilibrium

Einstein equations ↓

3 Intermediate and late time solution

←

4 Relaxation and hydrodynamic regime: thermalization and isotropization of QGP

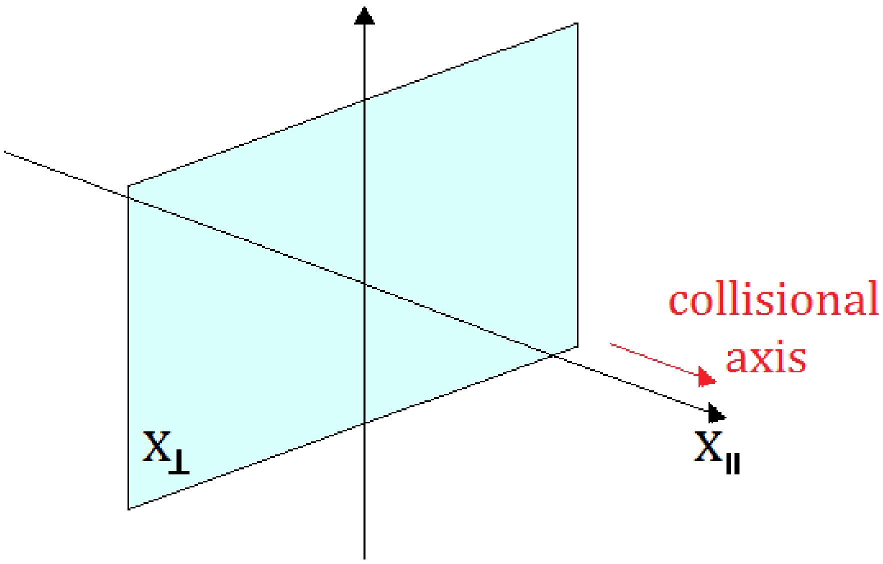
Simplification

Space-time symmetries

- Translation and rotation invariance in the $x_\perp$ plane

- Boost invariance along the $x_\parallel$ direction

Approximately realized at the central part of the QGP



Moreover...

"Local thermal equilibrium" : expansion is much slower than relaxation

All the portions of the fluid share the same (time dependent) temperature.

Boundary sourcing and QGP evolution

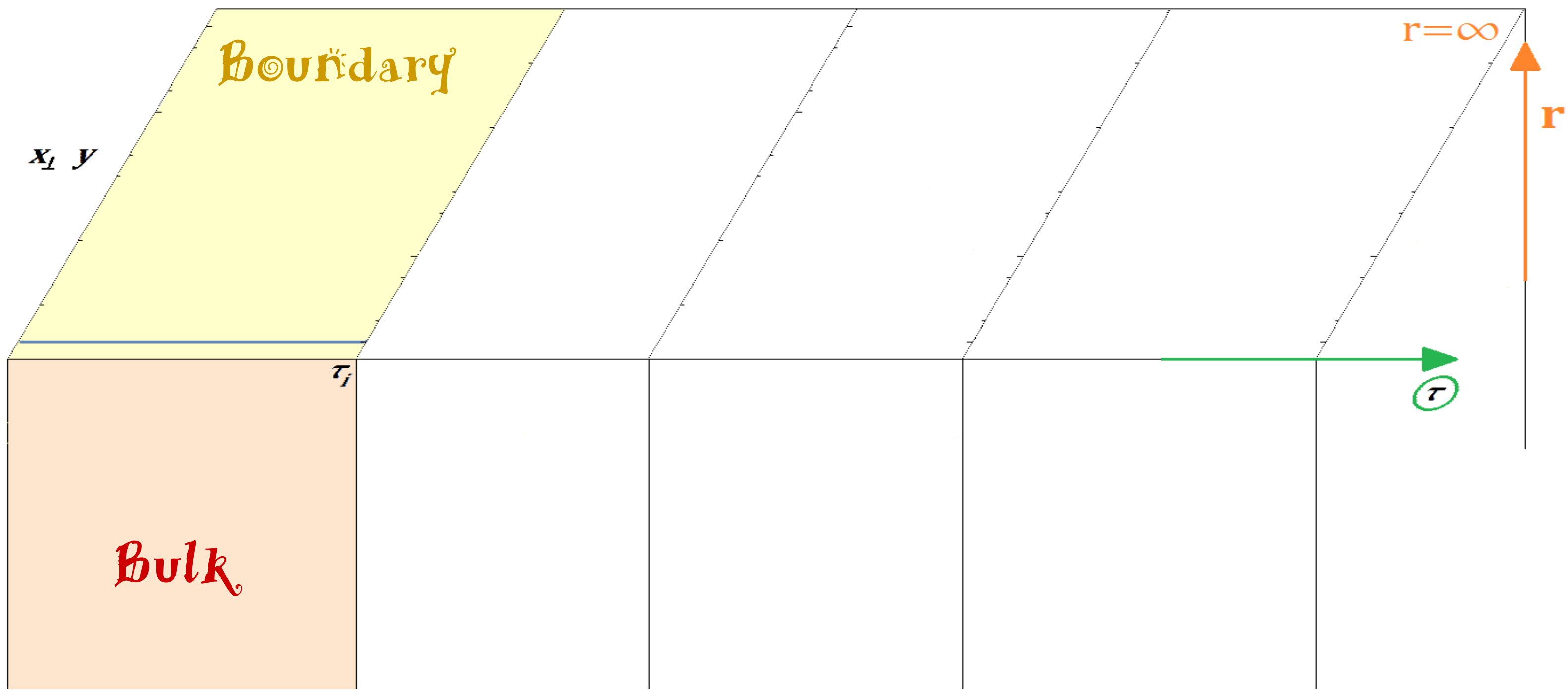
Natural coordinates for boost invariance: $(\tau, \boldsymbol{x}_\perp, y)$

$\begin{cases} x^0 = \tau \cosh y \\ x_\parallel = \tau \sinh y \end{cases}$

rapidity

proper time

Fifth holographic coordinate: bulk radius r



GROUND STATE

$$\mathcal{M}_4: \quad ds^2 = -d\tau^2 + d\boldsymbol{x}_\perp^2 + \tau^2 dy^2$$

$$AdS_5: \quad ds^2 = r^2\left[-d\tau^2 + d\boldsymbol{x}_\perp^2 + \left(\tau + \frac{1}{r}\right)^2 dy^2\right] + 2\,dr\,d\tau$$

Boundary sourcing and QGP evolution

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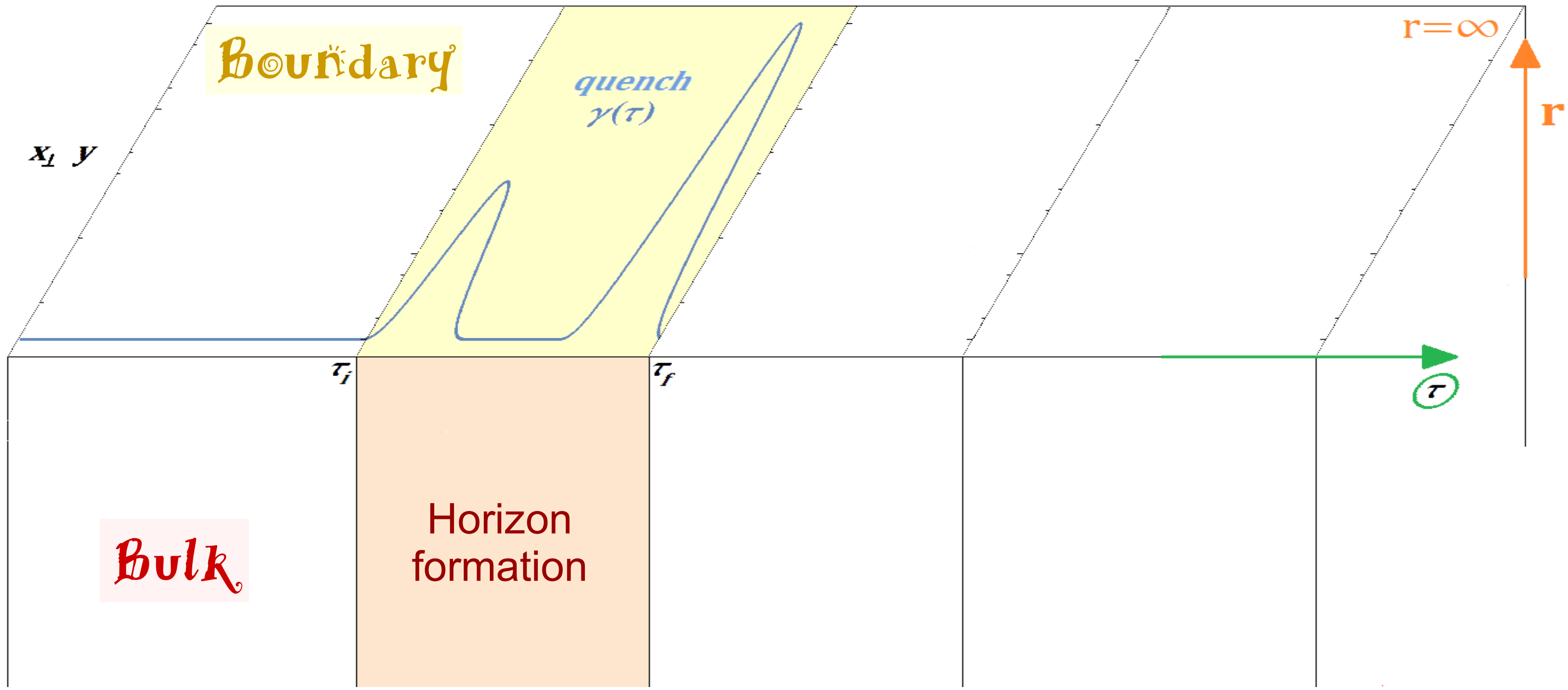


BOOST INVARIANT DEFORMATION

4d : $$ds^2 = -d\tau^2 + e^{\gamma(\tau)} d\boldsymbol{x}_\perp^2 + \tau^2 e^{-2\gamma(\tau)} dy^2$$

5d : $$ds^2 = -A(r,\tau)\, d\tau^2 + \Sigma^2(r,\tau)\left[e^{B(r,\tau)} d\boldsymbol{x}_\perp^2 + e^{-2B(r,\tau)} dy^2\right] + 2\, dr\, d\tau$$

Einstein's equations

Evolution in the 5-dimensional bulk

$A\,,B\,,\Sigma$ from Einstein's equations

$$\begin{cases}\Sigma\left(\dot{\Sigma}\right)' + 2\Sigma'\dot{\Sigma} - 2\Sigma^2 = 0\\ \Sigma\left(\dot{B}\right)' + \frac{3}{2}\left(\Sigma'\dot{B} + B'\dot{\Sigma}\right) = 0\\ A'' + 3B'\dot{B} - 12\frac{\Sigma'\dot{\Sigma}}{\Sigma^2} + 4 = 0\\ \ddot{\Sigma} + \frac{1}{2}\left(\dot{B}^2\Sigma - A'\dot{\Sigma}\right) = 0\\ \Sigma'' + \frac{1}{2}B'^2\Sigma = 0\end{cases}$$

$\Rightarrow$

- **Event horizon:** the critical geodesics $r_h(\tau)$ such that
$$\lim_{\tau\to\infty} A(r_h(\tau),\tau)=0$$
- **Apparent horizon:** from
$$\dot{\Sigma}(r_h(\tau),\tau)=0$$

Effective temperature and entropy density

Directional derivatives :

$f' \equiv \partial_r f$ along infalling radial null geodesics

$\dot{f} \equiv \partial_\tau f + \frac{1}{2}A\,\partial_r f$ along outgoing radial null geodesics

Boundary sourcing and QGP evolution

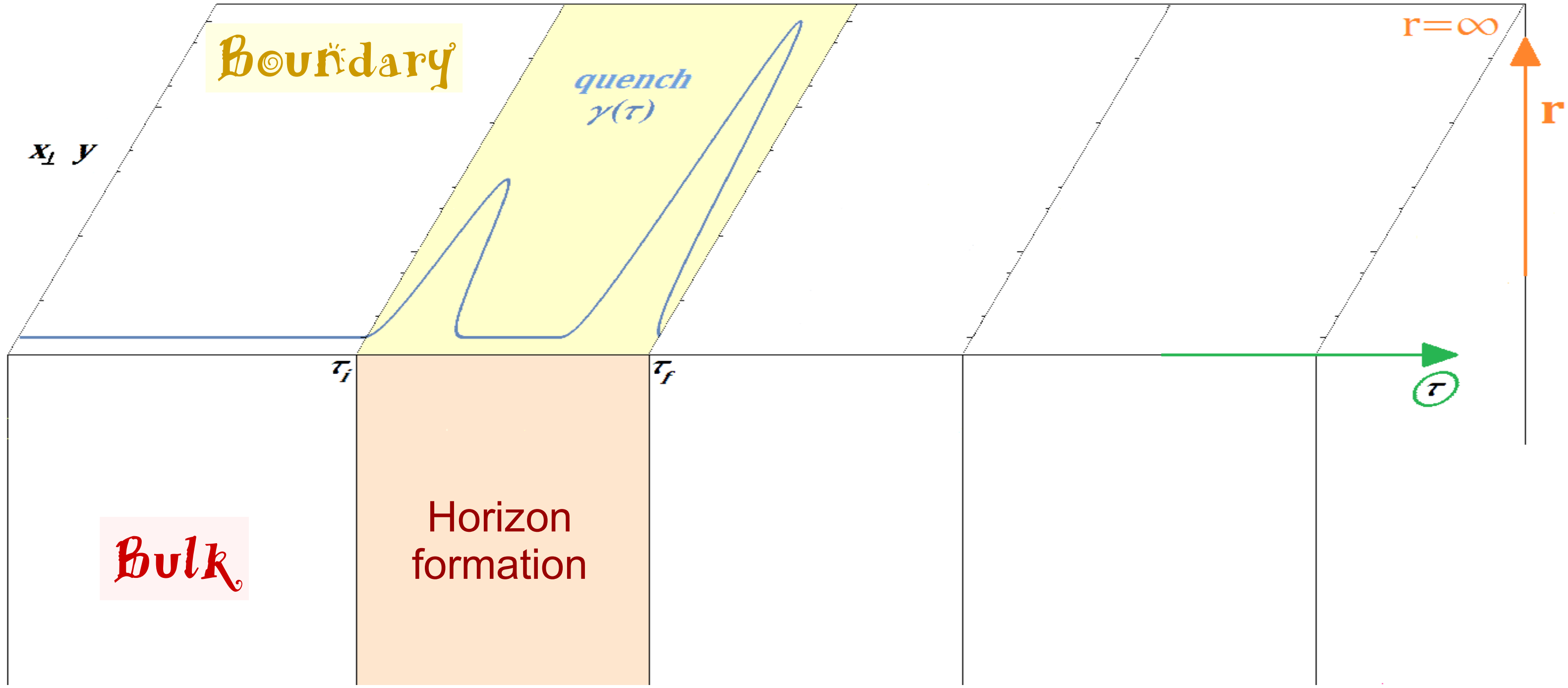
Natural coordinates for boost invariance: $(\tau, \mathbf{x}_\perp, y)$

y → rapidity

τ → proper time

$$\begin{cases} x^0 = \tau \cosh y \\ x_\parallel = \tau \sinh y \end{cases}$$

Fifth holographic coordinate: bulk radius r



BOOST INVARIANT DEFORMATION

4d : $ds^2 = -d\tau^2 + e^{\gamma(\tau)} d\mathbf{x}_\perp^2 + \tau^2 e^{-2\gamma(\tau)} dy^2$

5d : $ds^2 = -A(r,\tau) d\tau^2 + \Sigma^2(r,\tau)\left[e^{B(r,\tau)} d\mathbf{x}_\perp^2 + e^{-2B(r,\tau)} dy^2\right] + 2\, dr\, d\tau$

Boundary sourcing and QGP evolution

Natural coordinates for boost invariance: $(\tau, \boldsymbol{x}_\perp, y)$

τ → proper time; y → rapidity

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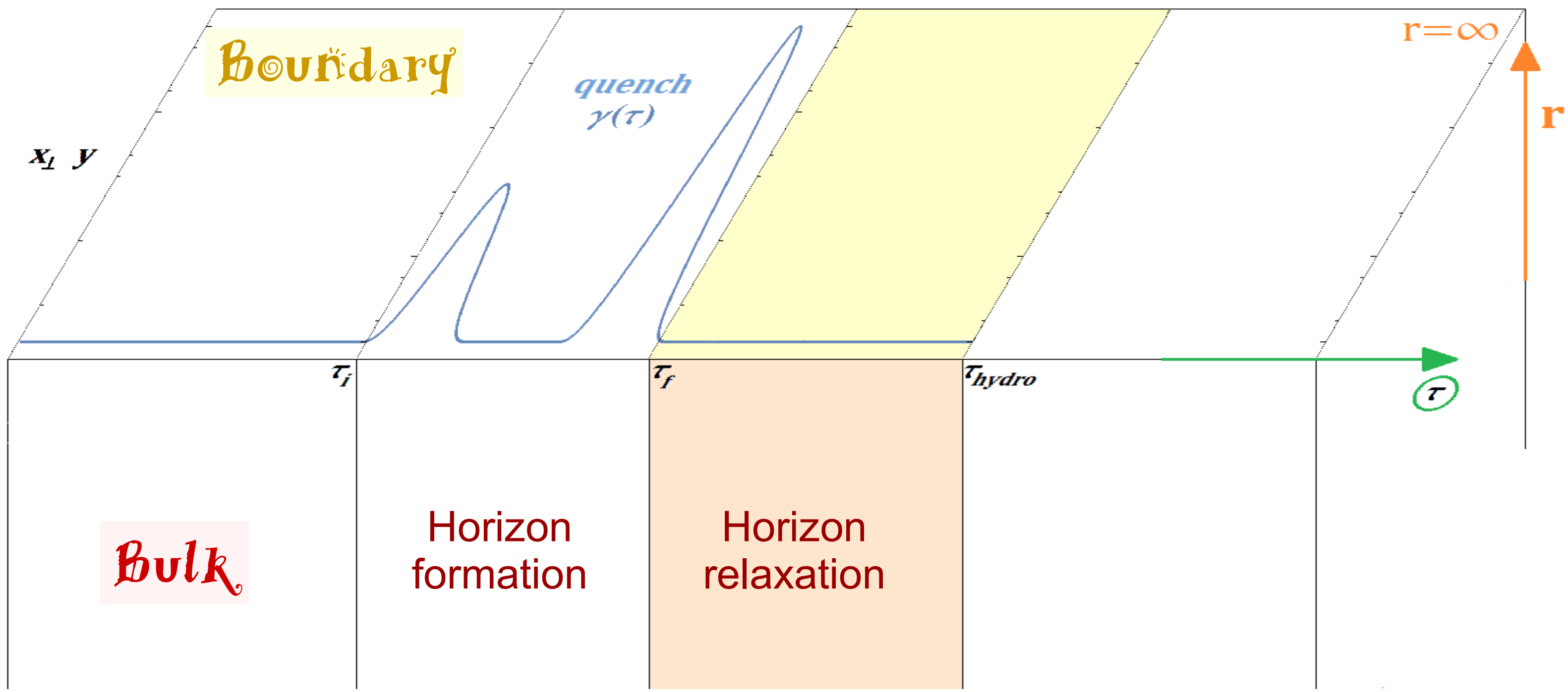


$$\tau_f \le \tau \le \tau_{\mathrm{hydro}}$$

THERMALIZATION and ISOTROPIZATION of the far-from-equilibrium state after the quench

Boundary sourcing and QGP evolution

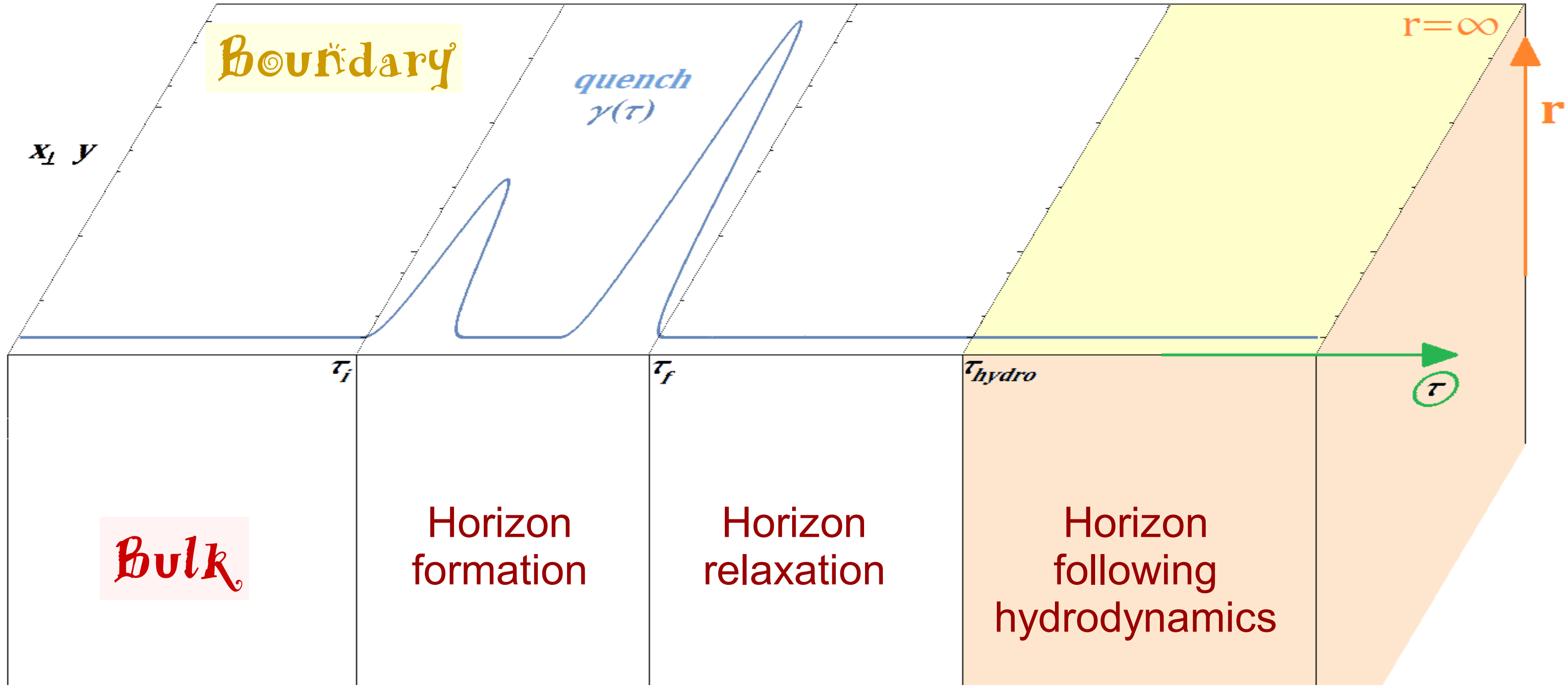
Natural coordinates for boost invariance: $(\tau, \boldsymbol{x}_\perp, y)$

$$\begin{cases} x^0 = \tau \cosh y \\ x_\parallel = \tau \sinh y \end{cases}$$

rapidity

proper time

Fifth holographic coordinate: bulk radius r



$\tau \geq \tau_{\text{hydro}}$

FINAL HYDRODYNAMIC REGIME: both temperature and stress-energy tensor follow hydrodynamics

Temperature and entropy density

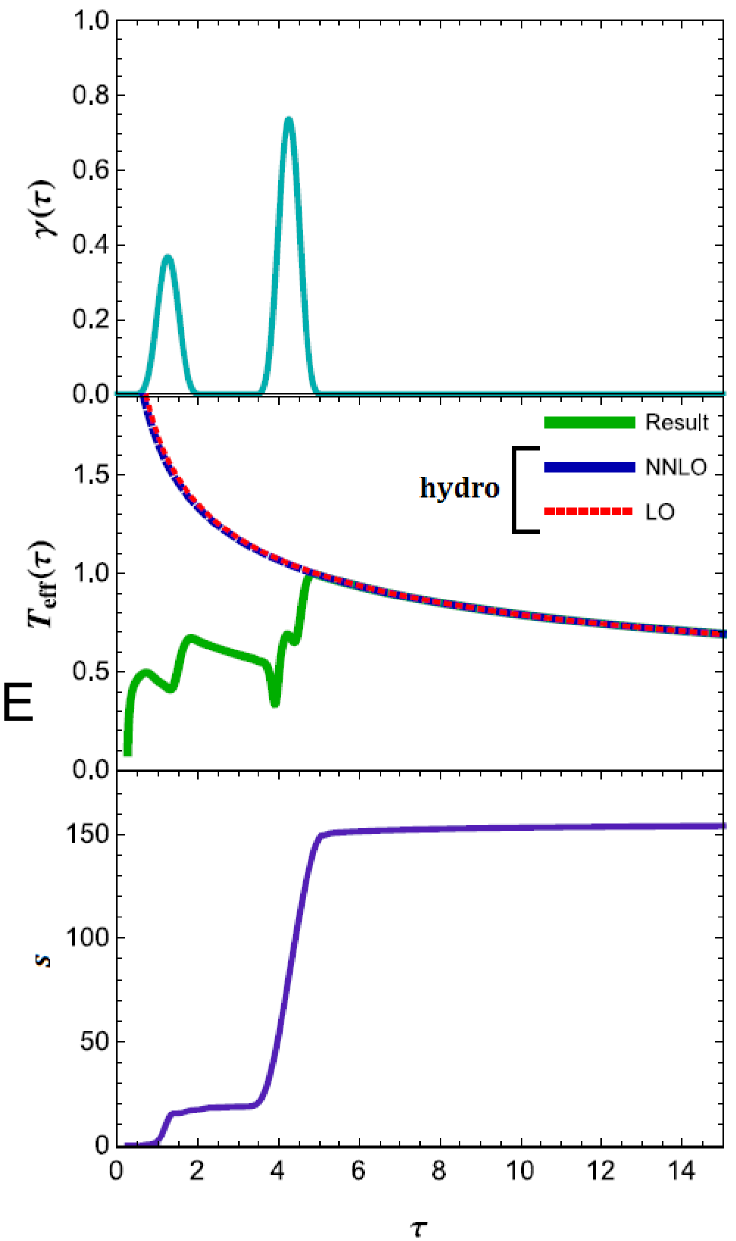
Boundary

Quench $\gamma(\tau)$

Bulk

Computed horizon $r_h(\tau)$

- follows the distortion profile
- asymptotically relaxes as $r_h(\tau) \propto \tau^{-1/3}$



BH THERMODYNAMICS + HOLOGRAPHIC PRINCIPLE

allow to define

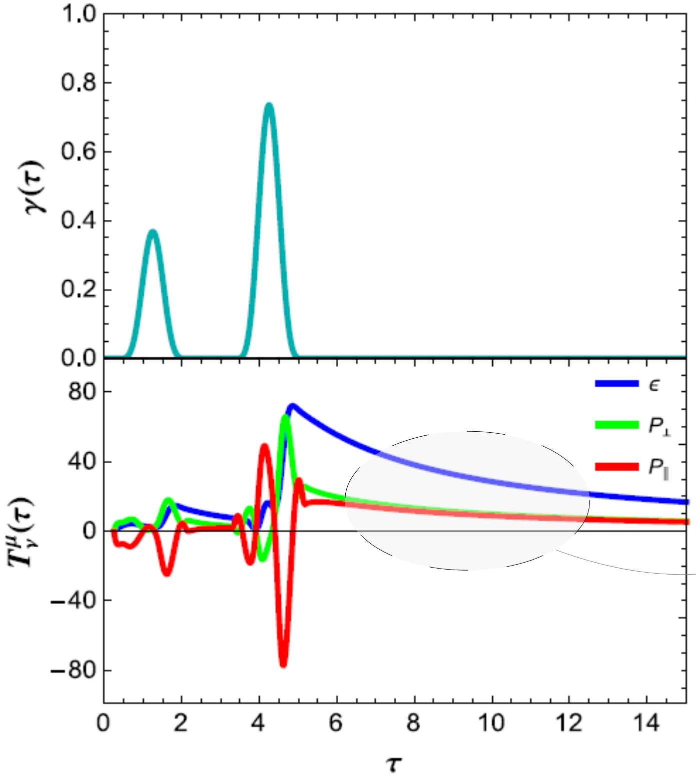
- Effective temperature from horizon position

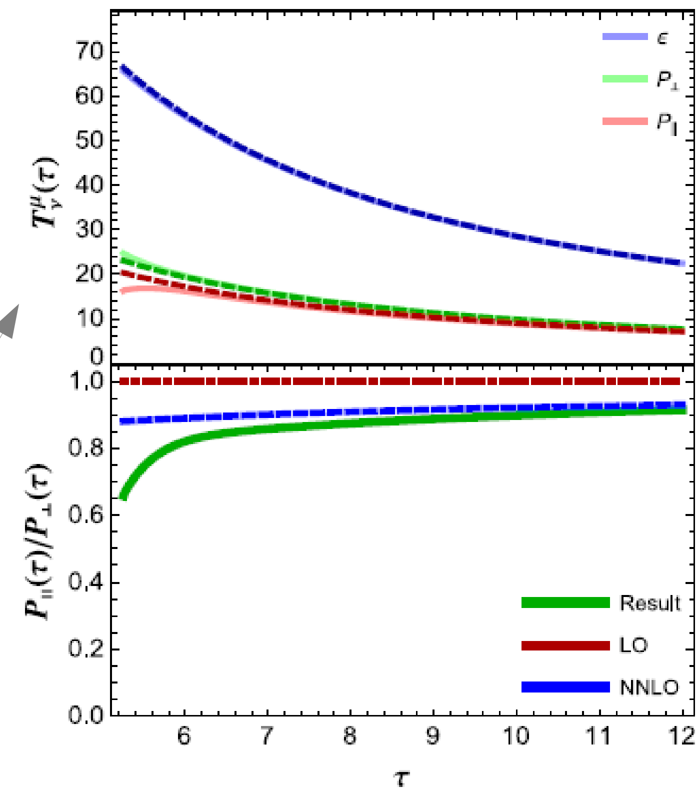
$$T_{\rm eff}(\tau) = \frac{r_h(\tau)}{\pi}$$

- Entropy from horizon area (Bekenstein-Hawking definition)

Energy density and pressure

Boundary Stress-Energy Tensor $T_{\mu\nu} = \frac{N_c^2}{2\pi^2}\,\mathrm{diag}\left(\mathcal{E}, \mathcal{P}_\perp, \mathcal{P}_\perp, \mathcal{P}_\parallel\right)$





Energy density $\mathcal{E}(\tau) = \frac{3}{4}\pi^4 T_{\mathrm{eff}}(\tau)^4$ starts to follow hydrodynamics as soon as the quench is switched off $(\tau = \tau_f)$.

Setting the scale $T_{\mathrm{eff}}(\tau_f) = 500\ \mathrm{MeV}$, pressure isotropy is reached after a time $\tau_{\mathrm{hydro}} - \tau_f \simeq 0.6\ \mathrm{fm}/c$.

Conclusions and perspectives

- The evolution dynamics of a boost-invariant non Abelian plasma has been analyzed in the holographic picture. The plasma is taken out-of-equilibrium by introducing a perturbation (quench) to the Minkowski boundary.

- We find that hydrodynamization is a two-step process: the temperature and the energy density acquire the hydrodynamical form as soon as the quench is switched off, while pressure isotropy is restored with a time delay of O(fm/c) [scale $T_{\mathrm{eff}}(\tau_f)=500\,\mathrm{MeV}$].

- Comparable estimates of the time delay are obtained for different quench profiles.

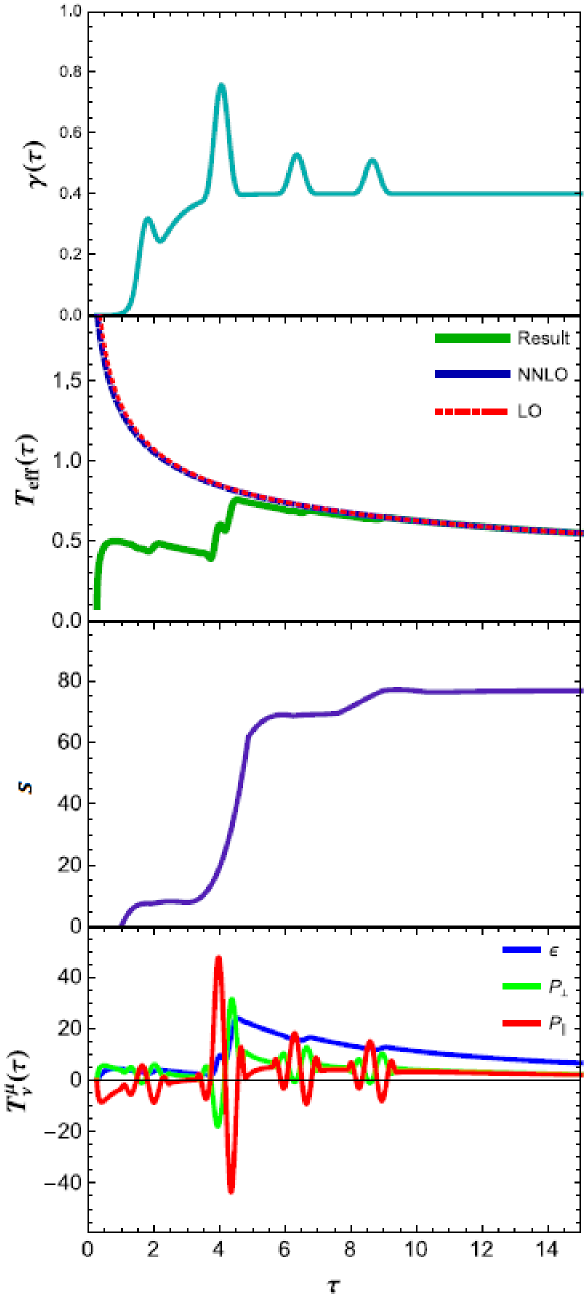
- Although the boundary sourcing is a rather theoretical representation of the process of colliding heavy ions, these results suggest us what can be expected in realistic situations.

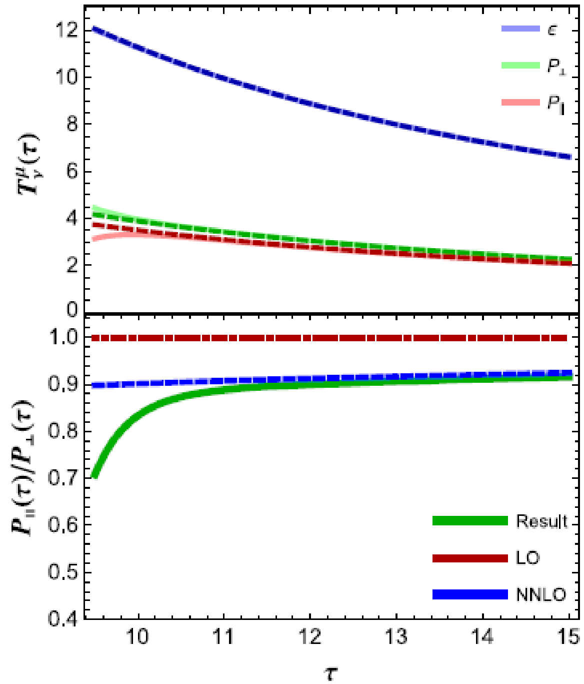
- **What next?** It should be possible to generalize this method in order to study more demanding problems which have less symmetry.

Thank you!

Bonus material

Other case studies





$\gamma(\tau) =$ a sequence of short pulses $+$ a smooth step

Very different time scales

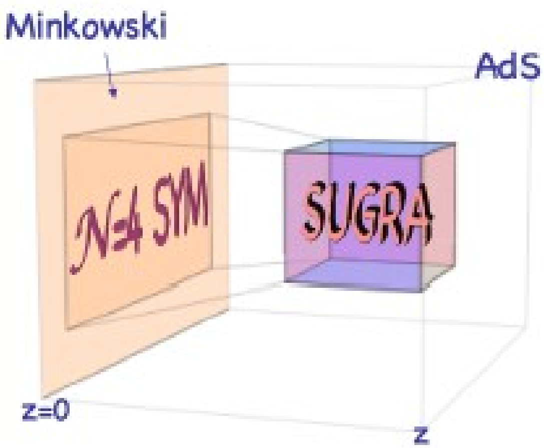
$T_{\rm eff}(\tau)$ and $\mathcal{E}(\tau)$ reach the hydrodynamic regime at $\tau=\tau_f$.

Pressure becomes isotropic after a time $\tau_{\rm hydro}-\tau_f \simeq 0.3\ \mathrm{fm}/c$ (scale $T_{\rm eff}(\tau_f)=500\ \mathrm{MeV}$).

Maldacena's conjecture

SUPER-YANG-MILLS

(SYM) on $\mathcal{M}_4$



IIB STRING on

$$AdS_5(R)\times \mathbf{S}^5(R)$$

- Coupling constant g_{YM}
- N=4 SUSY generators
- Gauge group $SU(N)_{\text{color}}$

Strong-coupling limit

$$N\to\infty,\quad \lambda=g_{YM}^2 N\to\infty,\quad g_{YM}^2\to 0$$

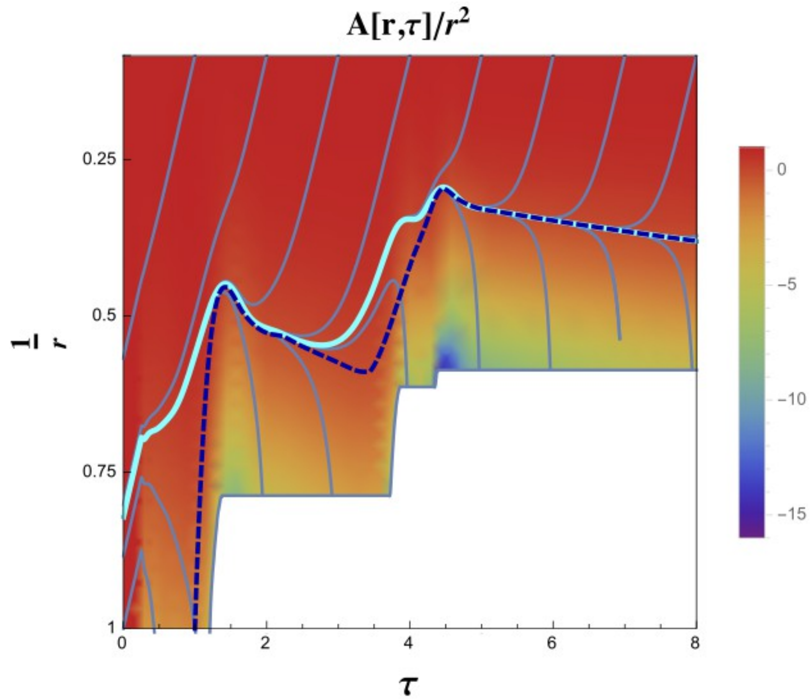
Maldacena

- Coupling constant g_s
- R curvature radius
- $\sqrt{\alpha'}$ length of the string

Supergravity limit

$$g_s\to 0,\quad R^2/\alpha'\to\infty$$

Apparent and event horizon



- The gray lines are radial null outgoing geodesics $\frac{d\,r}{d\tau} = \frac{A(r,\tau)}{2}$;
- The dashed dark blue line is the apparent horizon from $\dot{\Sigma}(r_h(\tau),\tau)=0$;
- The continuos cyan line is the event horizon obtained as the critical geodesics $r_h(\tau)$ such that $\lim_{\tau\to\infty} A(r_h(\tau),\tau)=0$.

Testing the numerical algorithm

$$\mathcal{R}_6 = \frac{\Sigma(\dot{\Sigma})' + 2\Sigma'\dot{\Sigma} - 2\Sigma^2}{|\Sigma(\dot{\Sigma})'| + |2\Sigma'\dot{\Sigma}| + |2\Sigma^2|}$$

$$\mathcal{R}_7 = \frac{\Sigma(\dot{B})' + (3/2)(\Sigma'\dot{B} + B'\dot{\Sigma})}{|\Sigma(\dot{B})'| + (3/2)(|\Sigma'\dot{B}| + |B'\dot{\Sigma}|)}$$

$$\mathcal{R}_8 = \frac{A'' + 3B'\dot{B} - 12\Sigma'\dot{\Sigma}/\Sigma^2 + 4}{|A''| + 3|B'\dot{B}| + 12\,|\Sigma'\dot{\Sigma}|/|\Sigma^2| + 4}$$

$$\mathcal{R}_9 = \frac{\ddot{\Sigma} + (1/2)(\dot{B}^2\Sigma - A'\dot{\Sigma})}{|\ddot{\Sigma}| + (1/2)(|\dot{B}^2\Sigma| + |A'\dot{\Sigma}|)}$$

$$\mathcal{R}_{10} = \frac{\Sigma'' + (1/2)B'^2\Sigma}{|\Sigma''| + (1/2)|B'^2\Sigma|}$$